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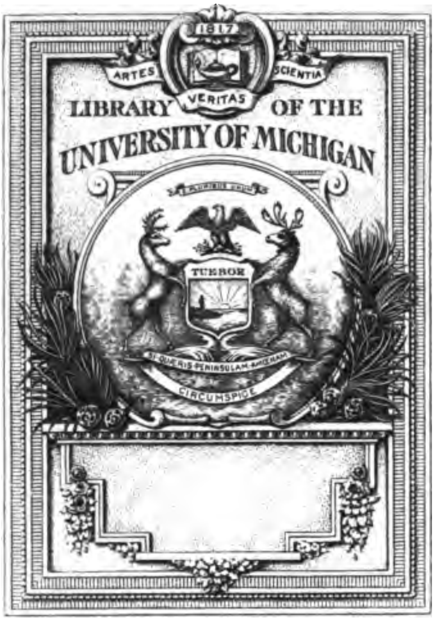

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GEOMETRY,

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PLATE I.

To deſcribe a Parallel.

See Fig. 1.

DRaw A B, and ſtrike two equidiſtant Arches C and D, from whoſe utmoſt extent, draw E E, which is parallel to A B.

N. B. Theſe Lines cannot meet, if extended to Infinity.

To cut a given Right Line thro' the Middle at Right Angles.

See Fig. 2. Draw G H, from whence ſtrike the Arches I I and K K to any equal Diſtance from their Centers, more than half the given Line, from whoſe Interſections draw L L which cuts G H at right Angles through the middle in M.

N. B. A right Line is ſtraight, and a right Angle is that which contains ninety Degrees of a Circle, commonly called a Square, or Perpendicular through the middle of a given Line.

To deſcribe a Right Angle at the End of a given Line.

See Fig. 3. Draw N O, and from O make five equal Diviſions of any Scale, and ſtrike the Arch 4 P at pleaſure, more than ninety Degrees; then extend O 5, and from 3 ſtrike the Arch Q to interſect 4 P, at R draw R O; then is the Arch 4 R 90 Degrees of a Circle, and the Lines N O and O R make a right Angle.

Demonſtration; Becauſe 3 R is equal to O 5, and O R to O 4, the Sides of the Angle are 3, 4, and 5, which are Root Numbers; the Square of the two leſſer being equal to the Square of the greater; their Root muſt be 5 and bears the ſame Proportion with each other, as 6, 8 and 10; for a further Demonſtration, ſee the 47th of the firſt Book of *Euclid*

PLATE II.

Another Way to deſcribe a Right Angle at the End of a given Line.

See Fig. 4. Lay down an equilateral Triangle of any Scale, as TVWT, and continue the Side

V W

Fig. 1.

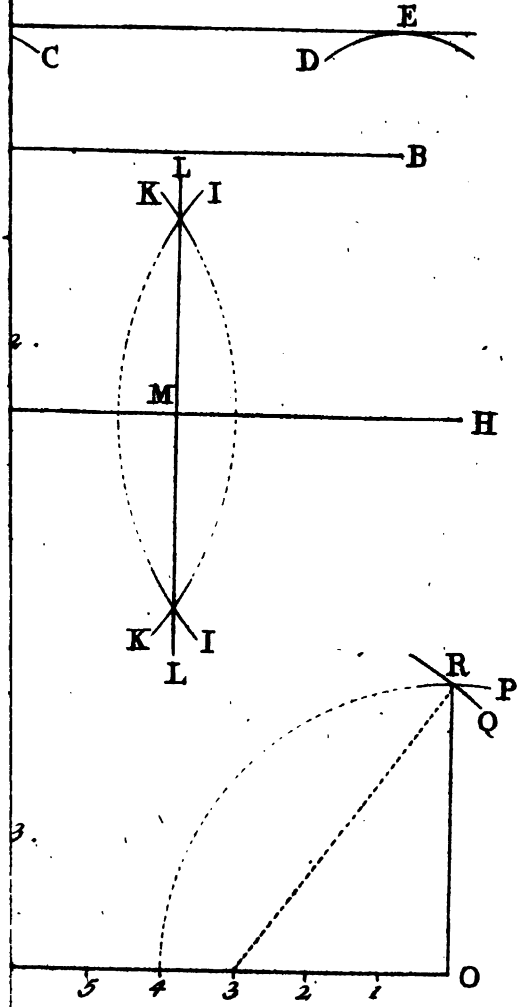
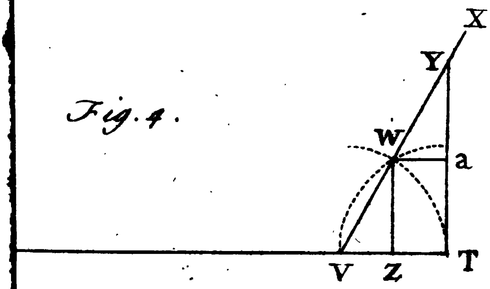


Fig. 4.



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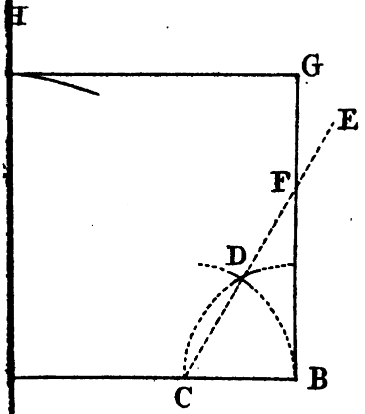

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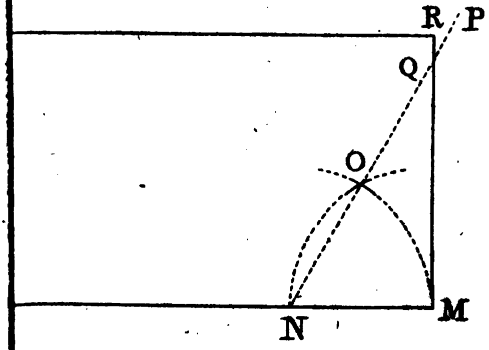


V W at pleaſure to X; extend the Side V W and ſet it from W to Y; then draw the Line Y T, which muſt make a right Angle with the given Line S T; becauſe the Triangles V W Z and W Y *a* are equal, and the Baſes V Z and W *a* are equal to V T.

N. B. An equilateral Triangle is that which has three equal Sides, as T V W T; See the 24th Definition of the firſt Book of *Euclid.*

To lay down a Quadrate, or equal-ſided right-angled plain Superficial.

See Fig. 5. Draw the Side AB and deſcribe the equilateral Triangle BC DB, of any Scale; continue the Side CD at pleaſure to E, and extend CD and ſet it from D to F; then is ABF a right Angle (becauſe D is over the middle of CB, therefore F muſt be over B, and conſequently in a right Angle with AB) then thro' F draw the Side BG, and extend AB or BG, and ſtrike the Arches H and I, which interſects at K, from whence draw the Sides K A and KG.

N. B. To find the ſuperficial Area of this Figure, multiply one Side in itſelf, the Product is the true Content.

To lay down a Parallellogram, or long Square, call'd a right-angled plain Superficial, whoſe Sides are unequal to the Ends.

See Fig. 6. Draw the Side LM, and deſcribe the equilateral Triangle MNOM, continue NO at pleaſure to P, and ſet the Diſtance NO from O to Q, and draw the End M R through Q; then extend L M and ſtrike the Arch T, alſo M R, and ſtrike the Arch S, which interſects each other at V, and draw the Lines V R and V L.

N. B. To find the Superficial Area of this Figure, multiply one Side by one End, and their Product is the true Content.

PLATE III.

All Angles are equal to half the Area of a Quadrate, or Parallellogram, whoſe oppoſite Sides are equal to the Baſe and Altitude of the Angle contained therein, and the Area of all Angles is found by multiplying the Baſe by half the Altitude, or the whole Altitude by half the Baſe.

See Fig. 7. A E F C is a Quadrate, or 4 equal-ſided right-angled Figure, and A B C an Iſoceles Triangle, whoſe Baſe A C, and Altitude H B, are equal to the oppoſite Sides of the Quadrate.

To find the Area of the Triangle A B C, multiply A C the Baſe by H K half the Altitude,

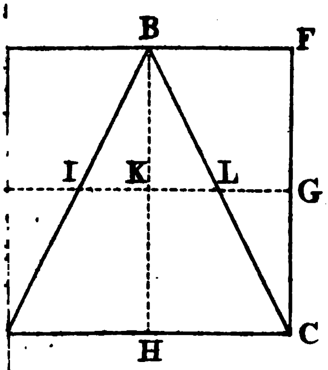


Fig. 7.

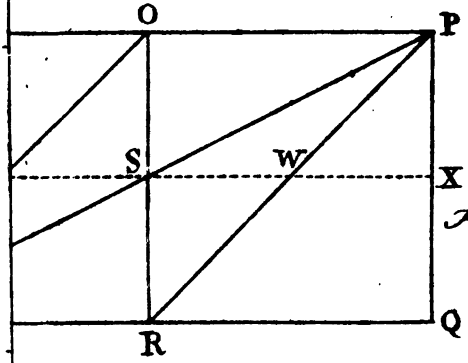


Fig. 8.

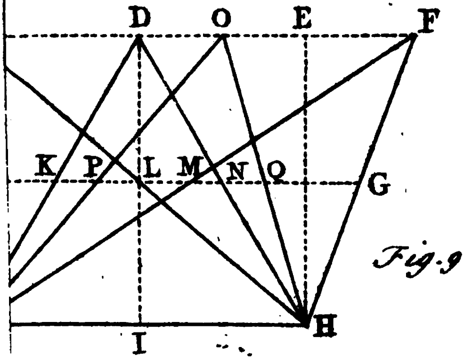


Fig. 9.

tude, becauſe the Angle B K I is equal to I D A, and B K L to L G C, the true Area muſt be equal to half the Quadrate A D G C. Again multiply the Altitude HB by HC half the Baſe; becauſe the Angle BEA is equal to BHA, and B F C to B H C; the Area of the Angle A B C muſt be equal to H B F C, half the Quadrate.

N. B. All Angles which hath two Sides equal, as A B and C B are term'd Iſoceles Triangles.

All ſuperficial Figures ſtanding on the ſame Baſe, and between, and the ſame Parallels are equal in themſelves. *See Fig.* 8.

Demonſtration. MNOR and ROPQ are two Quadrates equal to each other, which makes the Parallellogram M N P Q; the Rhomboides MOPR is equal to the Quadrate M N O R, becauſe the Angle R O P is equal to M N O; alſo O R M to P Q R. Again; the Angle M R O is equal to half the Quadrate R M N O, and the Angle M R P is equal to half the Rhomboides M O P R; therefore as the Rhomboides and Quadrate are prov'd to be equal to each other, the Angles M O R and M P R muſt be equal. See the 41ſt of the firſt Book of *Euclid.*

To find the Area of the Angles M R O, multiply the Baſe M R by R S half the Altitude, or the Altitude R O by half the Baſe M R, and their Content is the true Area. Again; To find the Area of the Angle MPR, multiply the Baſe M R by Q X half the Altitude, or Q P the Altitude by half the Baſe M R, and their Content is the true Area.

Again; The Rhomboides MOPR is equal to the Quadrate M N O R, becauſe O P is equal to N O, and V W to T S. Again; the Triangles are all equal, becauſe TV, VS, SW and WX are all equal, as is more fully ſhew'd in the following Figures.

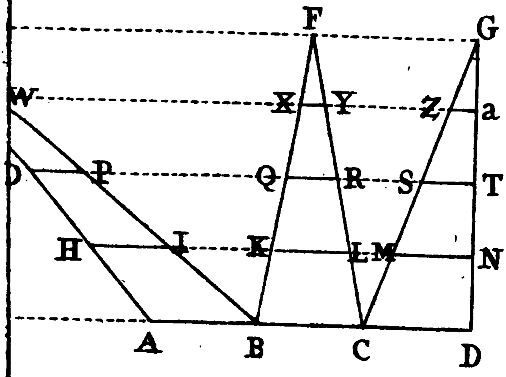
N. B. All Figures whoſe oppoſite Sides and Angles only are equal, as M O P R, are term'd Rhomboides. See the 33d Definition of the firſt Book of *Euclid.*

See Fig. 9. ACFH is a Trapezia, in which is deſcribed four different Angles, whoſe Areas are equal; H A C is a right Angle Triangle, A D H an Iſoceles Triangle, A O H a ſcalenon Triangle, and A F H an ambligonian Triangle, whoſe Areas are all equal, and B L, K N, P Q and M G are equal. For a farther Demonſtration ſee the following Example.

N. B. All four ſided Figures, whoſe Sides are not equal, are term'd Trapezia's. See the 34th Definition of the firſt Book of *Euclid.*

P L A T E IV.

See Fig. 10. The Areas of the Triangles A E B, B F C and C G D are equal; becauſe the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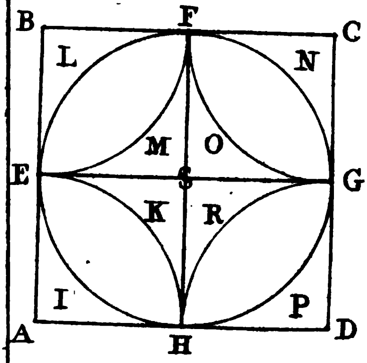
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Fig. 11.

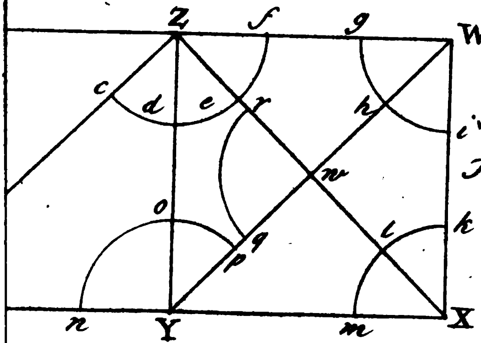
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Fig. 22.

the parallel Lines $AB=BC=CD$; $HI=IK=KL=MN=OP=QR=ST$ and $VW=XY=Za$ are equal in their reſpective Situations, and ſo are all parallel Lines in Figures of equal Baſe and Altitude, to Infinity. *See* Fig. 10.

Of the Quantity of Angles.

As the Arch of a Circle is divided into 360 Parts, call'd Degrees; two Lines drawn from the Center, to any of that Number, are call'd the Quantity of the Angle, created by thoſe Lines.

See Fig. 11. ABCD is a Quadrate, in which a Circle is deſcribed, HF and EG are right angular Diameter Lines, and HIELFNGPH the Circumference; therefore the Angles HSE, ESF, FSG, and GSH, muſt be four Right Angles, whoſe Quantities are each 90 Degrees, as HIE, ELF, FNG, and GPH, becauſe 4 Times 90 is 360. Again, as the Quantities of the four Right Angles, ſpringing from S, contains 360 Degrees, ſo does thoſe ſpringing from ABCD, becauſe the Curve HIE is equal to EKH, *&c.* which proves, that the Angles of four Lines, touching each other, muſt contain 360 Degrees. For a farther Demonſtration, ſee the following Example.

See Fig. 12. TVWX is a Parallellogram divided into two Quadrates, by the Line YZ; the Angles *s*T*u*, *a*V*b*, *g*W*i*, and *k*X*m*, are equal right Angles, and conſequently contains 360 Degrees. Again, T Z W Y is a Rhomboides, whoſe Angles *p* Y *n* and *c* Z *f* are equal and obtuſe, containing each 135 Degrees, and the Angles *s* T*t* and *g* W*b* are equal and acute, containing each 45 Degrees, add the 4 Quantities together, which are 360. Again, T Z *w* Y T is a Trapezia, whoſe Sides are not equal, *s* T *t* is an acute Angle of 45 Degrees, *c* Z *e* and *r w q* are right Angles of 90 Degrees each, and *p* Y *n* 135 Degrees, whoſe Quantities added together amounts to 360 Degrees. Again, all Triangles contain 3 inward Angles, whoſe Complements added together amounts to 180 Degrees, which is equal to two right Angles; Z Y T Z is a right angled Triangle; *s* T *t* and *c* Z *d* are acute Angles of 45 Degrees each, and O Y *n* is a right Angle of 90 Degrees, added together amounts to 180 Degrees.

N. B. A Triangle is the whole ſuperficial Area, contain'd within three Lines meeting each other, as T Z X T, which was demonſtrated by *Fig.* 9. but an Angle is two Lines meeting, which incloſes no Area; thoſe which form a Square, as W X Y, are term'd a right Angle, and if inclining leſs than a Square, an acute Angle, as Y W X; but if declining more than a Square, an obtuſe Angle, as T Z W.

PLATE V.

To find the Area of a Trapezia, which contains two Right Angles, and has two oppoſite Sides parallel.

See Fig. 13. Multiply A B, or D G, by the Middle F I, their Content is the true Area of the Trapezia A B C E, becauſe C D I is equal to I G E. Again, add the oppoſite Sides A E and B C together, and multiply half their Length by A B, or G D, the Content is the ſame; becauſe C D is equal to G E, therefore B C and A E added together, muſt be twice the Length of the middle Line F I.

To find the Area of a Trapezia, which has no Right Angles, but has two oppoſite Sides parallel.

See Fig. 14. Multiply S Q, or T R, by the middle Line L O, their Content is the true Area of the Trapezia K M N P, becauſe S K L is equal to L Q M, and N R O to O P T. Again, add M N and K P together, and multiply half their Length by S Q, or T R, their Content is the ſame, becauſe K S= Q M=N R and T P, are equal; therefore M N and K P added together, muſt be twice the Length of the middle Line L O.

N. B. The Meaſure of all Dimenſions muſt be taken at right Angles from each other, otherwiſe the Content will not be true. For Example, ſee *Fig.* 14, if K M was multiplied by L O, their Content would be ſo much more than Truth, as the Line Q R multiplied by the Difference between the Lines K M and S Q.

To find the Area of a Rhomboides.

See Fig. 15. Multiply the Side S Y, or T W, with S V, or Z W, their Content is the true Area of the Rhomboides T W Y S; becauſe the oppoſite Sides are equal, alſo the Angles S T V, W Z Y and Y Z W, therefore the Content muſt be true, and the Area of Y S T W equal to the Area of S V X Y.

To find the Area of a Trapezia, whoſe oppoſite Sides are neither parallel, nor equal.

See Fig. 16. A B C D is the Trapezia, whoſe Area is required, and A *g e f* is a Parallellogram, whoſe Sides touches the ſeveral Angles, the former has no oppoſite Sides parallel; and bears no demonſtrable Proportion with the latter, therefore cannot be meaſured as the three foregoing Examples.

To find its Area, divide it into two Triangles, by the Line A C, which is called the Baſe, common to both, and draw Lines from

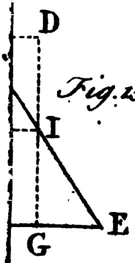


Fig. 13.

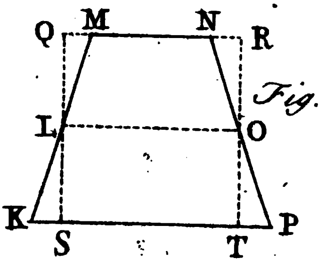
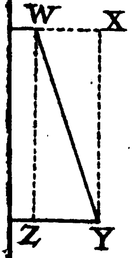


Fig. 14.



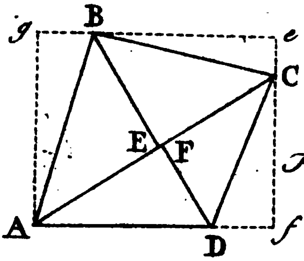
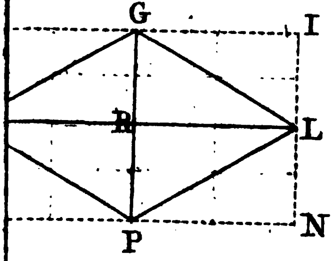


Fig. 16.



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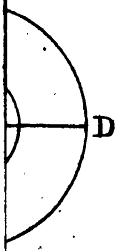


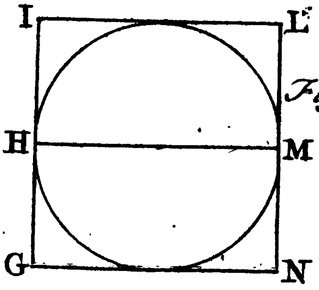


Fig. 19.

D and B to E and F, at right Angles with A C; then add the Lines B E and F D together, and multiply half their Length by the Baſes A C; the Content is the true Area of the Trapezia A B C D.

See Fig. 17. A diamond-like Figure, whoſe Sides are all equal, is term'd a Rhombus, as CGLP. See the 32d Definition of *Euclid*, Book I.

To find the Area of a Rhombus.

C G L P is the Figure inſcribed in the Parallellogram A E I N, whoſe Area is required; the Triangles C G L and L P C are equal, and C L the Baſe common to both, and P R and G R their Perpendiculars equal to each other, and at right Angles with the Baſe C L.

Multiply the Baſe C L, by half the Perpendiculars P R and G R, and their Content is the true Area; becauſe P A C is equal to C R P = C E G to G R C = G I L to L R G and L N P to P R L, therefore the Area of the Rhombus C G L P is equal to half the Area of the Parallellogram A E I N; which proves that half the Length of two Perpendiculars, multiplied with one common Baſe, gives the Area of both Angles.

OF CIRCLES.

THE true Demonſtration of the Area of a Circle has been attempted by many, but without Succeſs; neither is it poſſible they ſhould ſucceed, till an Affinity is found, between a Curve and a ſtraight Line; yet there is an approved practical Rule laid down by our Fore-Fathers, to ſquare the Circle, ſufficient to meaſure the moſt valuable Material, as in the following Example.

See Fig. 18. The Lines A D and E F are term'd the Diameter, and the Curve AFDE the Circumference: The one is found by practical Experience to be to the other as 7 is to 22; for Example, if AD be any given Length, ſuppoſe 10, what is the Curve A F D E? Work by the Rule of Proportion and ſay, if 7 gives 22, what does 10 give? the Quotient is the Length required.

To find the Area, multiply the Circumference A F D E by A B one fourth of the Diameter, their Content is the Area; or half the Diameter, as A C, by half the Circumference F D E, their Content is the ſame.

Demonſtration. A Circle is ſuppoſed to contain an infinite Number of Triangles, (as the Quadrant A F C) tending to the Center C, whoſe Baſe is the Circumference AFDE, and their Perpendicular half the Diameter A C;

A C; therefore by the foregoing Rule for the meaſuring of Triangles, A B being half A C, multiplied by the Circumference A F D E, muſt be the Area requir'd. Again, as A C half the Diameter is the Perpendicular, and A F D half the Circumference, or Baſe; they multiplied together muſt produce the ſame Content.

Of the Proportion between a Circle and Quadrate, whoſe Diameter and Sides are equal.

See Fig. 19. H M is the Diameter of a Circle, whoſe Length is 14, and G I L N a Quadrate whoſe Sides are each 14.

Firſt find the Content of the Circle by the Rule laid down to *Fig.* 18. whoſe Content will be 154. Then ſquare the Quadrate, whoſe Content will be 196; divide 154 by 14, the Quotient will be 11, alſo divide 196 by 14, the Quotient will be 14, which proves the Circle to be $\frac{11}{14}$ths of the Quadrate, therefore the Area of a Circle may be found thus.

Square the Quadrate, whoſe Content multiply by 11 and divide the Product by 14, the Quotient will be the Area of the Circle. Again, to know the Diameter of a Circle, which ſhall contain any given Number; multiply the given Number by 14, and divide their Product by 11, the Square Root of the Quotient is the Diameter required.

PLATE VI.

To deſcribe an Ellipſis, whoſe Area ſhall be equal to any given Part of a Circle.

See Fig. 20. A B C D is a Quadrate with a Circle inſcribed therein, and I K L M is a Parallellogram with an Ellipſis inſcribed, whoſe Area bears the ſame Proportion to the Parallellogram, as the Area of the Circle does to the Quadrate, and the Area of the Ellipſis is equal to half the Area of the Circle; becauſe all the parallel Lines within the Ellipſis are equal to thoſe without, as H P = P Q = Q O and O G, and the Curve E O F P, is found by dividing all the parallel Lines in the Circle into four equal Parts, in which Diviſions tack Nails and bend a thin Lath about them, and ſtrike the Curve.

To find the Area of the Ellipſis E O F P, multiply the tranſverſe Diameter E F by the conjugate Diameter O P, and their Product by 11 and divide by 14, the Quotient is the Area: becauſe the Ellipſis bares the ſame Proportion to the Parallellogram, as the Circle does to the Quadrate.

N. *B*. If the Area of the Elipſis is requir'd to be equal to $\frac{1}{3}$ of the Circle's Area, the parallel

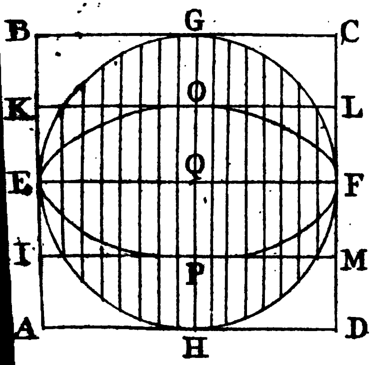


Fig. 20.

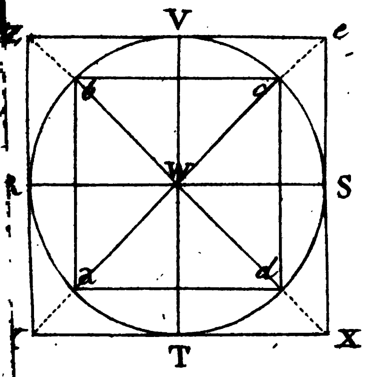


Fig. 21.

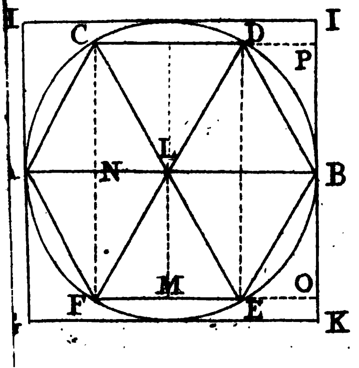


Fig. 22.

rallel Lines in each Semi-Circle, muſt be divided into three equal Parts, and ſo of any other Proportion.

Of the proportionable Area, between a circumſcribed and inſcribed Quadrate to a Circle, alſo of the Areas of the four Segments.

See Fig. 21. Y Z, ze, ex and X Y, are each 14; the Circle R V S T is $\frac{11}{14}$ths of the Quadrate $yzex$, and the Quadrate $abcd$ is $\frac{7}{14}$ths or half, and the Segments $cbvc$, &c. are each $\frac{1}{14}$th Part of the outer Quadrate; firſt the Circle is proved to be $\frac{11}{14}$th of the outer Quadrate by *Fig.* 19, and the inner Quadrate is $\frac{7}{14}$ths or half the outer; becauſe the Diameter ac is equal to the Diameter R S, and the Semi-diameter W b is equal to the Semi-diameter W V: therefore abc and cda are two equal Triangles, ac multiplied by wb muſt give the Area of the leſſer Quadrate, $abcd$, and R S multiplied by WV, muſt alſo give the Area of half the greater Quadrate, as R Z es equal to the leſſer Quadrate. Again, becauſe the Circle is $\frac{11}{14}$ths of the greater Quadrate, and the leſſer Quadrate $\frac{7}{14}$ths; ſubſtract 7 from 11, and $\frac{4}{14}$ remains for the four Segments C bv C, *&c.* which proves the Segments to be each $\frac{1}{14}$th of the greater Quadrate, and $\frac{1}{11}$th of the Circle; ſo is the Segment of all Quadrants to their whole Circle. Again, ſuppoſe the Quadrant W a T d W of any Radius, it is $\frac{1}{4}$ of its Circle, and conſequently 2 and $\frac{3}{4}$ of 11 Parts; its Segment *a* T *d a* has already been proved to be $\frac{1}{11}$, therefore its Triangle *a w d a*, muſt be 1 and $\frac{3}{4}$ of 11 Parts of the Circles.

This is done without knowing the Sides of the Quadrate *a b c d*, which may be found thus. Extract the Root of its Product, and the Quotient is the Length of the Side *a b*, &c. or by the 47th of the firſt Book of *Euclid* add the Square of *d w* to the Square of W C, their Root is the Side *c d*.

Of an Hexagon, or Figure of ſix Sides, and equal Angles.

See Fig. 22. The ſeveral Sides of this Figure AC=CD=DB=BE=EF and FA, are each equal to the Semi-diameter A L, *&c.* becauſe 6 Times 60 is 360, and that Number the Quantity of Degrees in a Circle; therefore the Quantity of the ſix Angles meeting at L, muſt be each 60, and as the Quantity of the three Angles, in all Triangles, contain 180 Degrees, (as is proved by *Fig.* 12, *&c.*) L C and L D being equal, C D muſt be the ſame and the Quantities of the Angles at L C and D each 60 equal to 180; for the ſame Reaſon if LC and LD be continued to any equal Length, CD will continue equal diſtant with the ſame, and the Quantity of the Angles at C and D, each 60, equal to that of L;

which proves three Angles of 60, muſt produce an equilateral Triangle, and that a Hexagon is naturally compoſed of ſix. For a farther Demonſtration, ſee the 15th of the 4th Book of *Euclid*.

If the Side FE of a Hexagon be given, and the other Sides are required, deſcribe the equilateral Triangle E F L, and from L ſtrike the Circle E F A C D B, and with the ſame Extenſion, ſet on the Circle the other five Sides.

To find the Area of a Hexagon, firſt know the Length of the Perpendicular L M, by a Rule; or thus, from the Square of F L, ſubſtract the Square of F M, the ſquare Root of the Remainder is the Length L M; half of which multiply by the Baſe F E, and their Product by 6, the Content is the Area; or multiply the Diſtance F C, by $\frac{3}{4}$ of the Diameter as N B, the Product is the ſame; becauſe the inward Angles C N A and F N A are equal to the outward Angles, D P B and E O B. Obſerve, that Lines drawn between the Points CE and DE, divides the Diameter A B into four equal Parts.

PLATE

PLATE VII.

To deſcribe a Pentagon, or five-ſided Figure, one Side being given.

See Fig. 23. (But take no Obſervation of the Triangle A H B, nor of the continued Lines between 3 K and F H, till they are mentioned in the Demonſtration) A B is the Side given, from B ſtrike the Arch A C at pleaſure, and raiſe the Perpendicular B D, divide the Arch AD into five equal Parts, and ſet one of thoſe Parts on the Arch from D to E, and draw the Side B E; then is the Arch A E divided into Six equal Parts; draw the Line B 3, then middle the Line A B, as at I, from whence raiſe a Perpendicular to touch the Line B 3, as at F, which is the Center of the Circle A G H E B, which deſcribe, and ſet on the Sides A G=G H and H E.

Demonſtration. Firſt remember that the Quantities of the Angles in all three-ſided Figures are 180 Degrees (as was proved by *Fig.* 12) and the Example being a five-ſided Figure, is divided into three Triangles, *viz.* A G H=A H B and B H E, whoſe Quantities added together are 540, and is the Quantity contained in all five-ſided Figures. Again, the Quantity of the Angles A B E is 108th, which is $\frac{1}{5}$ of 540, becauſe A B D is a right Angle

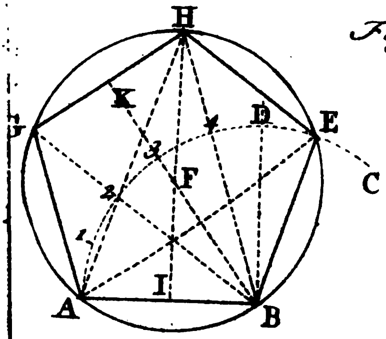


Fig. 23.

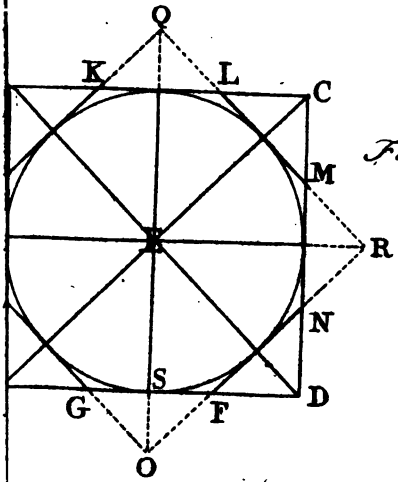


Fig. 24.

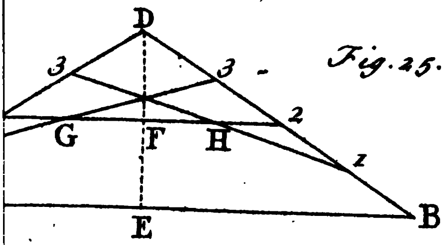


Fig. 25.

Angle conſequently 90, and DE 18, the $\frac{1}{5}$ added thereto muſt be 108.

To find the Center of the Circle AGHEB; obſerve, A 3 and E 3 are equal, and the Line B 3 cuts the Quantity of the Angle ABE in the middle, which if continued to K, will cut the Side GH in the middle, and a Perpendicular from I touching the Line B 3, muſt give the Center F, and if continued, will cut the Point H. Again, IH is equal to BK; FB to FH, and FI to FK, &c.

To deſcribe a Pentagon in a given Circle at H, make an Angle whoſe Quantity muſt be 36 Degrees, and continue the Sides as to A and B; then will AB be equal to one of the Sides, whoſe Extent is equal to each of the other 4 Sides, and AHB is an Iſoceles Triangle, whoſe Quantities at A and B are double the Quantity at H, and a Line drawn through the middle of the Quantity of the Angle ABH, cuts the Point G, and the ſame from A, gives the Point E. See the 11th of the 4th Book of *Euclid.*

To find the Area of a Pentagon, multiply one Side, as AB, by half IF, and their Product by 5 gives the true Area.

To deſcribe an Octagon, or eight-ſided Figure, on a Quadrate, ſee *Fig.* 24, (but make no Account of the four dotted Angles, till mentioned in the Demonſtration) ABCD is the given Quadrate, and AC and DB its Diagonals; extend AE and ſet it from A to

I and F: alſo B E and ſet it from B to H and L; again, ſet C E from C to K and N; alſo D E and ſet it from D to G and M; then is A I and A F=BL and B H=CK and C N D G and D M; all equal to A E, &c. half the Diagonal; then deſcribe the Sides G H=I K=L M and N F which are all equal to the Sides H I=K L=M N and F G, and G H I K L M N F G is the Octagon required.

Demonſtration. Becauſe the Quadrate A B C D is equal to the reverſe Quadrate O P Q R, and they being laid down at right Angles with each other, plainly proves that all their different Lines and Angles muſt be equal in themſelves.

To find the Area of an Octagon, multiply one Side, as G F, by half the Perpendicular S E, and their Product by 8 gives the true Area.

N. B. The Circle in this Figure is deſcribed for no other Uſe than to ſhow the equal Diſtance of the Sides from the Center E.

Of Tracery, which are practical Curves, created by ſtraight Lines.

To deſcribe a Scheme Arch.

SEE Fig. 25. Draw the given Baſe A B, which middle as at E, and raiſe a Perpendicular to D, twice the Heighth of the intended Arch; then draw the Lines D A and D B, and divide each into the ſame Number of equal Parts (the more the truer the Curve, as will be ſeen in the next Example, theſe being divided into no more than four for the ſake of Demonſtration) and draw Lines from 1 to 3, from 2 to 2, and from 3 to 1, which gives ſix Angles, or Points of a Curve, as at A I G H I B.

Demonſtration. The Perpendicular E D muſt be twice the Heighth of the intended Curve, becauſe a Line drawn thro' the middle of the two oppoſite Sides of any Triangle, will cut the Perpendicular of that Angle thro' the middle; therefore a Line drawn through the Diviſions 2 2, in the oppoſite Sides of the Angle A D B, muſt cut the Perpendicular E D in the middle, as at F, and it will be more fully proved by the next Example, that no one of any Number of Lines, drawn through equal Diviſions on the oppoſite Sides, will cut below the Point F, provided they are drawn through their proper correſponding Diviſions, as from 1, 3=2, 2=3, 1, &c.

PLATE VIII.

See Fig. 26, whoſe out Lines are equal to thoſe of the foregoing Example. The oppoſite Sides AE and EB are each divided into twelve equal Parts, and each interſecting Line is drawn between the equal Numbers, as from 1 to 1, 2 to 2, *&c.* and the Line 6,6, cuts the Perpendicular CE in the middle at D.

To find the Center of a ſcheme Arch, or part of a Circle, the Baſe and Perpendicular being given. See *Fig.* 27. FG is the Baſe, which middle at M and raiſe the Perpendicular MH, which let fall in a direct Line at pleaſure to I; draw FH, then from H, at any Extent, ſtrike the Arch KL, and with the ſame Extent from F ſtrike the Arch NC at pleaſure; then take the Radius KL and ſet it on the Arch NC, as from N to O, and thro' O draw the Line FP, to interſect the Line HI, as at Q, which is the Center of the Curve FHG.

Demonſtration. As the Line HI interſects the middle of the Baſe, or Cord-Line FG, at right Angles, it muſt of Conſequence paſs through the Center of the Circle, of which the Curve FHG is a part. Again, H and F being

Fig. 26.

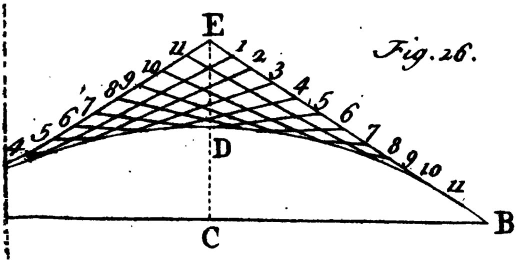
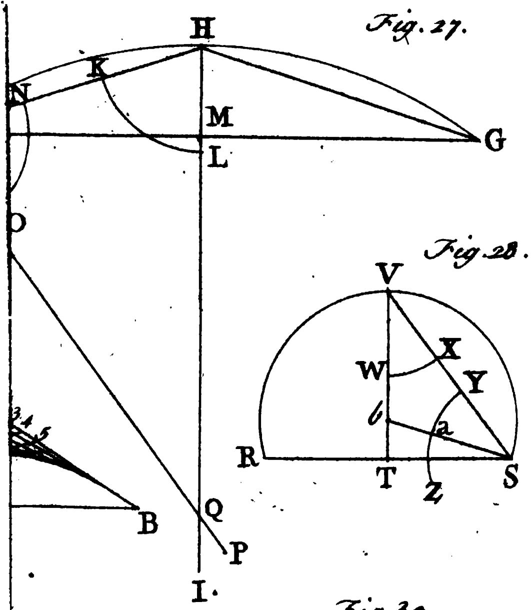


Fig. 27.

Fig. 28.



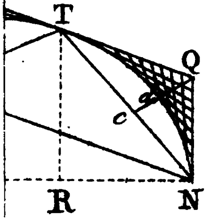
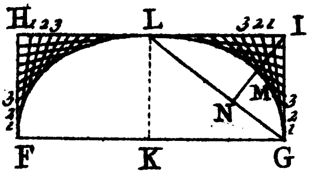


Fig. 30.



being both Points on the Circle, and F H I an Angle, whoſe Quantity is K L, and its Side H I cuts through the Center, therefore the Side F P, of the Angle H F P, whoſe Quantity N O is the ſame, will alſo cut thro' the Center; and their Interſection as at Q, muſt be the Center of the Circle; as alſo of its Part F H G.

To find the Center for Part of a Circle; whoſe Area is more than a Semi; the Baſe and Perpendicular being given.

See Fig. 28. R S is the given Baſe, which middle as at T, and raiſe the Perpendicular T V; then draw the Diagonal S V, and with any Extent from V; ſtrike the Arch W X; with the ſame Extent from S, ſtrike Y Z at pleaſure; then extend the Radius W X and ſet it on the Arch from Y to *a*, and draw a Line from S through *a*, to interſect the Line T V as at *b*; which is the Center of the Arch R V S.

Demonſtration. As the Perpendicular raiſes from the middle of the Baſe R S, it naturally paſſes through the Center of the Circle, of which R V S is a part (as in the foregoing Example) becauſe the Quantities W X and Y *a* are equal, the Angle S V T and V S *b* are equal, and as the Side V T of the Angle S V T paſſes thro' the Center, the Side S *b* of the Angle V S *b* will alſo paſs thro' the Center,

ter, and their Interſections as at *b*, muſt be the Center of the Circle, of which R V S is the greater Part. Again, becauſe *b* V and *b* S are equal, and meet at the point *b*, they are each equal to the Semi-Diameter of the Circle, becauſe no other Lines, drawn from the Curve of a Circle, can meet in the diameter Line as T V, but thoſe which are equal to the Semi-Diameter of the Circle.

To deſcribe a Curve in an Obtuſe, or Scalenon Triangle, by Interſections of Lines.

See Fig. 29. A C B is the Angle, and A E B is the Curve, whoſe Heighth D E, is equal to half the Perpendicular D C.

To deſcribe the Curve, divide A C and C B each into the ſame Number of equal Parts, and draw the interſecting Lines from 1 to 1, 2 to 2, and ſo on, as in *Fig.* 26.

To deſcribe an Ellipſis in a Parallellogram.

See Fig. 30. Draw the out Lines F H I G F, which middle by the Line K L, and divide F H and H L, alſo L I and I G, each into the ſame Number of equal Parts; then draw the interſecting Lines from 1 to 1, *&c.* which creates the Curve F L G, and N M is equal to M I.

To deſcribe a ramping Ellipſis.

See Fig. 31. Draw M N, which middle at R, and raiſe the Perpendiculars M O P, R T and N Q; then draw the Lines O N and P Q, parallel to each other, and divide the Lines O P=P T=T Q and Q N into the ſame Number of equal Parts, and draw the Lines of Interſection as in the foregoing Example, which gives the Curve *o b* T *d* N; *a b* is equal to *b* P, and *c d* to *d* Q.

PLATE IX.

To deſcribe an Oval, and Gothic Arch, by Centers.

See Fig. 32. Draw N O, which divide into four equal Parts, and ſtrike the three Circles; then interſect N O in the middle, by a perpendicular Line at pleaſure, as W and *d*; extend T V, and from T and V deſcribe the interſecting Arches at *d*, alſo from S and R to *c*; then is *c* and *d* the Centers of the Curves N T *g* V O and O S *h* R N, which compleats the Oval.

To deſcribe the Gothic Arch N *a* W *b* O, *ſuppoſe* N O *to be the Baſe.*

Draw a Line from Y, through the Center *e*, to cut the Arch N T as at *a*, alſo from X, thro'

thro' the Center *f*, to cut the Arch O V, as at *b*, and from Y ſtrike the Arch *a* W, and from X ſtrike the Arch *b* W, then *is* N *a* W *b* O a Gothic Arch, whoſe half Baſe N *g* and Perpendicular *g w*, are equal.

To deſcribe an Oval and Gothic Arch another Way.

See Fig. 33. Draw the tranſverſe Diameter A B, which divide into three equal Parts, and ſtrike the ſeveral Curves, from C deſcribe I A K and H D G, from A, K C I, from D, H C G and F B E, and from B, E D F; then from G ſtrike the Curve K O F, and from H ſtrike the Curve I P E, which deſcribes the Oval A K O F B E P I.

To deſcribe the Gothic Arch A N B, draw a Line from E, thro' C, to cut the Arch A K as to L; alſo from I, thro' D, to cut the Arch F B as at M; then from E ſtrike the Curve L N, and from I ſtrike the Curve M N, which compleats the Gothic Arch A L N M B.

Obſerve all mixt Curves are Parts of greater and leſſer Circles, and their Centers muſt correſpond in a ſtraight Line, as G C K and G F D, for the Ellipſis, &c. and as E C L and I D M, for the Gothic Arch; otherwiſe the Meeting of the ſeveral Curves as at K F. E I and L M will be crippled and their Correſpondence weak and offenſive to the Eye; and 'tis further to be obſerved, that

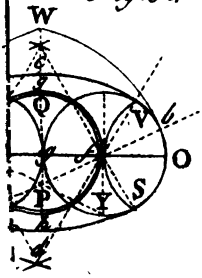


Fig. 32.

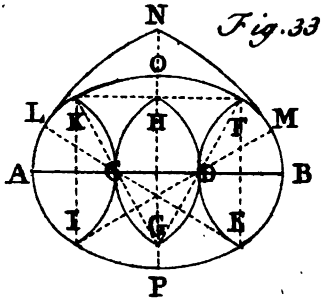


Fig. 33.

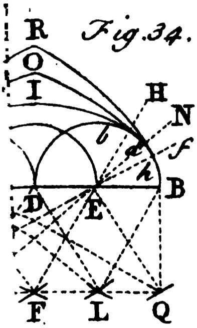


Fig. 34.

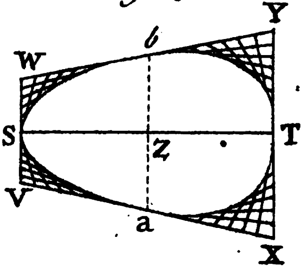


Fig. 35.

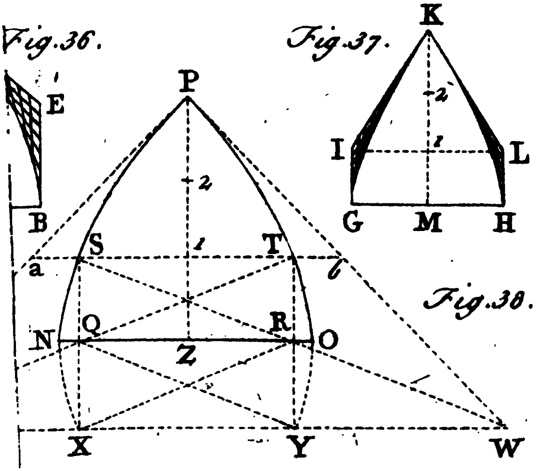


Fig. 36.

Fig. 37.

Fig. 38.

that this Manner of deſcribing Ovals is done by equilateral Triangles, as A K C and G K F, *&c.*

To deſcribe an Ellipſis, and two different Gothic Arches.

See Fig. 34. Divide the Baſe A B into four equal Parts, and ſtrike the three Semi-Circles; extend C E and draw the equilateral Triangle C E F, then draw correſponding Lines F G C and F E H, and F is the Center of the Curve A I B, which compleats the Ellipſis A *a* I *b* B.

To deſcribe the Gothic Arch A O B, draw two equilateral Triangles as A K D and D L B, and draw the correſponding Lines L C M and K E N; then from L ſtrike the Curve *c* O, alſo from K ſtrike the Curve *d* O, which compleats the Arch A *c* O *d* B.

To deſcribe the Gothic Arch A R B, draw the correſponding Lines Q C *e* and P E *f*, and from P ſtrike the Curve R *h*; alſo from Q the Curve *g* R, which compleats the Arch A *g* R *h* B.

To deſcribe the Curve of an Egg, by the Interſection of Lines.

See Fig. 35. The tranſverſe Diameter S T is divided into two equal Parts at right Angles in Z, by the conjugate Diameter *a b* and

PLATE X.

To deſcribe a Bow Arch by Centers, whoſe Heighth is equal to half its Baſe.

See Fig. 39. Draw the Parallellogram A C D B, which middle by E F, and draw the Triangle A F B, then is the Lines A F and F B the Baſes of two Curves, as A K F and F M B; and ſet on their ſpringing Heights, as L M, or I K; mark twice that Height on the Blade of a Square, as L H, or I G; and ſlide the Stock of the Square on the Line F B, till the Mark on the Blade touches the Line C D, as at H; draw L H, and make a Point as at M, and ſet up I K and K G in the ſame Manner, and draw G I; then continue H L at pleaſure to S, and G I at pleaſure to T; and where they cut the Line E F, as at N, is the Center for the Curve K F M; then from B and M ſtrike interſecting Arches, as at R and Q; alſo from A K to O and P, through R Q continue a Line to cut S H, as at V; alſo through P and O, to cut T G, as at W; then is V the Center for the Curve M B, and W the Center for the Curve A K, which compleats the Bow Arch A K F M B.

To

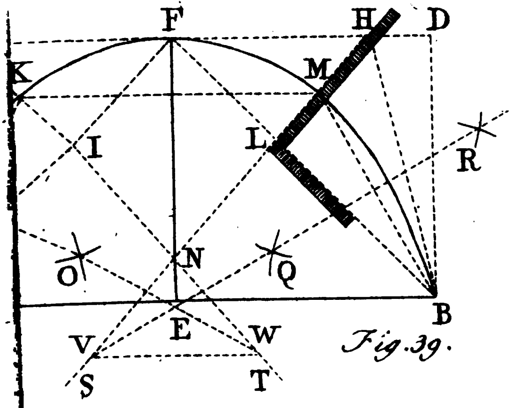


Fig. 39.

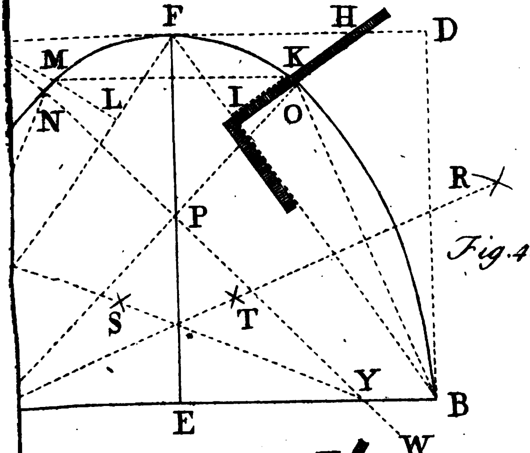


Fig. 40.

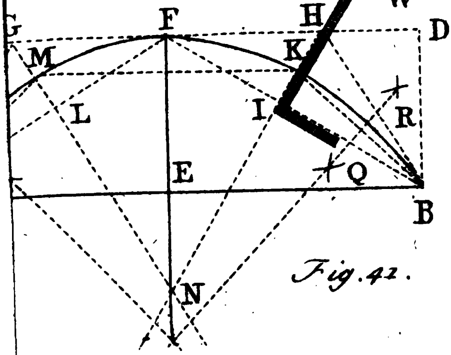


Fig. 42.

To deſcribe an Ellipſis Arch on its conjugate Diameter, to any given Heighth, by Centers.

See Fig. 40. Draw the Parallelogram A C D B A, which middle by E F, and deſcribe the Triangle A F B, whoſe Sides A F and B F, are Baſes of two Curves, as A N F and B O F, then aſſign the ſpringing Heighth of the Curves, as I K, or L M; which double and mark on the Blade of the Square, (as in the foregoing Example) and ſlide the Stock of the Square on the Line F B, till the Mark touches the Line C D, as at H; make a Point at K and H, alſo from the Line A F to L M, and G; then is M F K three Points of a Curve, whoſe Center muſt be found by interſecting Arches, from MF and K F, or by the Rule laid down in *Fig.* 27, which is ſuppos'd to be at P; draw a Line from H, through P, at Pleaſure to V, alſo from G, through P, at Pleaſure to W; then extend P M, and from P ſtrike the Curve NMFK O, and from O and B, ſtrike the interſecting Arches, as at T and R, alſo, from N and A, ſtrike thoſe at S and Q, and continue a Line through R and T to touch H V as at X; alſo, through Q S to touch W G, as at Y; then is X the Center of the Curve O B, and Y the Center of the Curve A N; which compleats the Ellipſis Arch A N M F K O B.

To deſcribe a Bow Arch, whoſe ſpringing Heighth is $\frac{3}{10}$ of its Spand.

See Fig. 41. Draw the Parallelogram ACDB, which middle by EF, and continue it down at Pleaſure towards N; then deſcribe the Triangle AFB, whoſe Sides AF and BF are the Baſes of two Curves, as AMF and FKB; aſſign the ſpringing Heighths LM, or IK; which double and ſet it on the Blade of the Square, (as in the foregoing Examples;) then move the Square along the Line BF, till the Mark touches the Line CD, as at H; make a Point at H and K, and by the ſame Method make a Point at G and M; then is MFK, three Points of a Curve, whoſe Center is N; draw a Line from H, through N, at Pleaſure, alſo from G through N at Pleaſure; and from B and K ſtrike interſecting Arches, as at Q and R; alſo from M and A, ſtrike thoſe at S and T; then through R and Q, draw a Line to touch the continued Line HN; alſo a Line through T and S, to touch the continued Line GN, and thoſe two Interſections, or Meetings, are the Centers of the Curves KB and MA, which compleats the Bow-Arch AM FKB.

PLATE

Fig: 42.

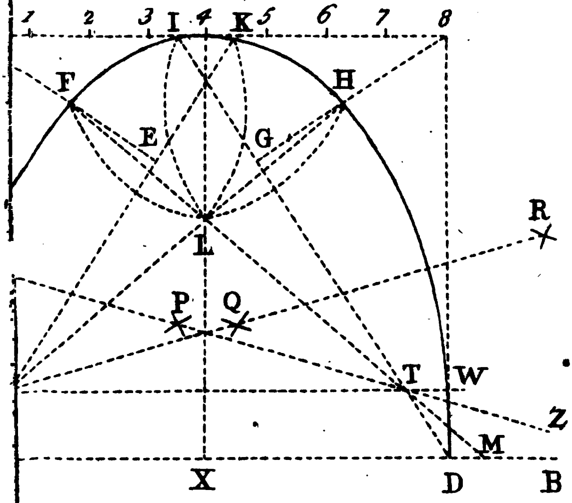

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Fig. 43.

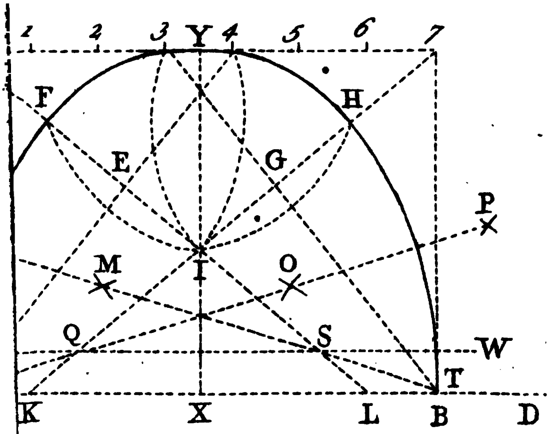
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PLATE XI.

To deſcribe an Arch, whoſe ſpringing Heighth is $\frac{7}{8}$ of its Spand.

See Fig. 42. Draw the Parallelogram C 7 8 D, and let the Sides C 7 and D 8, be $\frac{7}{8}$ of C D, or 7, 8; middle 3, 4 and 5, 4, as at I K, and draw C K and I D; then draw 7 E, and 8 G, at right Angles with C K and D I, and middle E 7, and G 8, as at F and H; extend H I, and from H ſtrike the Curve I L, from I ſtrike H L, from K ſtrike F L, and from F ſtrike K L; then is F K L and L I H the Points of two equilateral Triangles, and L is the Center of the Curve F 4 H; then draw a Line from F through L at Pleaſure as to M; alſo from H through L at Pleaſure as to N, and from C F H and D ſtrike interſecting Arches, as at O P Q and R; then draw the like R Q Y to touch the Baſe A B, and where it cuts H N at S, is the Center for the Curve H W, and the Meeting of thoſe Lines beyond A Y, is the Center of the Curve W D; again, draw the Line O P Z to touch the Baſe A B, and where it cuts F M at T, is the Center of the Curve V F, and the Meeting of the two Lines beyond B Z, is the Center of the Curve V C, which compleats the Arch C V F 4 H W D.

To deſcribe an Arch, whoſe ſpringing Heighth is $\frac{5}{7}$ of its Spand.

See Fig. 43. Draw the Parallelogram, (whoſe Sides A 5, and B 7, are each equal to $\frac{5}{7}$ of the Baſe A B or 5 7) which middle by X Y, and draw A 4, and B 3; then from 5 and 7, draw Lines to E and G, at right Angles with A 4, and B 3, which middle as at F and H; extend H 3, and from H ſtrike the Curve 3 I, from 3 ſtrike H I, from 4 ſtrike F I, and from F ſtrike 4 I; then is I F 4, and I H 3, the Points of two equilateral Triangles, and I is the Center of the Curve F Y H; then from F and H, draw Lines through I to L and K, and from A F H, and B, ſtrike interſecting Arches, as at N M O and P, and draw the Line P O R and N M T, and where they cut the Lines H K and F L, at S and Q are the Centers for the Curves H W, and V F, and R is the Center of W B and T, of the Curve V A, which compleats the Arch A V F Y H W B.

PATE XII.

To deſcribe a Cove, whoſe Projection is $\frac{7}{9}$ of its ſpringing Heighth.

See Fig. 44. Draw the intended perpendicular Heighth A 9, which divide into nine equal

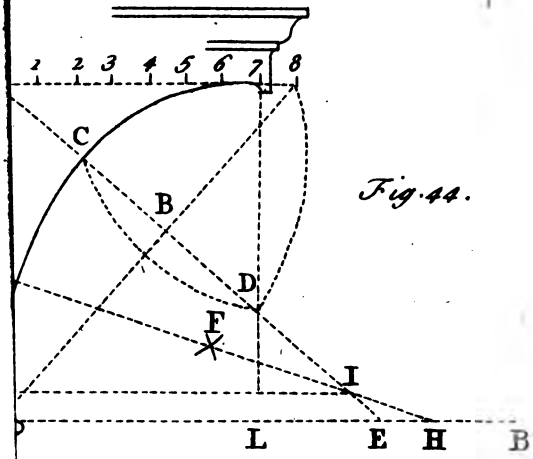


Fig. 44.

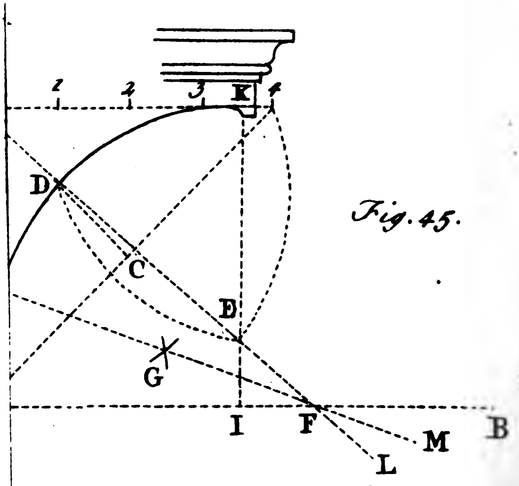


Fig. 45.

equal Parts, and let the Projection 9 7, be equal to 7 of thoſe Parts, and drop the Perpendicular 7 L, and add another Part from 7 to 8, and draw 8 A; then from 9 draw a Line to B at right Angles with A 8, and middle 9 B as at C; extend 8 C, and from C ſtrike the Curve 8 D, from 8 ſtrike the Curve C D, then is C 8 D the Points of an equilateral Triangle, and D is the Center of the Curve C 7, and from C, draw a Line through D to E; then from A and C ſtrike interſecting Arches, as to G and F, through which draw the Line G F H, and where it cuts the Line C E, as at I, is the Center of the Curve K C, and H is the Center of A K, which compleats the Curve A K C 7.

To deſcribe a Cove, whoſe Projection is $\frac{7}{8}$ of its ſpringing Heighth.

See Fig. 45. Draw the intended perpendicular Heighth A 4, and divide it into 4 equal Parts, and let 4 K be 3 and $\frac{1}{2}$ of thoſe Parts, and drop the perpendicular KI; then add half a Part from K to 4, and draw the Diagonal A 4, and from 4 draw a Line to C at right Angles with the Diagonal A 4, which middle as at D; then extend D 4, and from D ſtrike the Curve 4 E, and from 4 ſtrike the Curve D E; then is D 4 E, the Points of an equilateral Triangle, and E is the Center of the Curve D K; then from D draw a Line through E at

Pleaſure to L, and from A and D ſtrike interſectional Arches, as at H and G, through which draw H G M, and where it cuts the Line D L as at F, is the Center for the Curve A D; which compleats the Curve of the Cove A D K.

PLATE XIII.

To deſcribe three different Arches, whoſe Curves are mix'd like that of a Spring-Bow.

See Fig. 46, 47, and 48, whoſe Letters of Reference are all the ſame, and one Explanation demonſtrates the whole; draw AB the Baſe, which middle by C D at right Angles of a Length, at Pleaſure; then aſſign the ſpringing *a* E, and draw the Triangle AEB, who Sides A E and B E, are Baſes to the two Curves A I E and ENB; draw F G parallel to A B, and aſſign the ſpringing Heighth H I, or M N, which double and ſet on the Lines HK and M O, by a Square, as in *Fig.* 39, *&c.* which will give the Points I E N whoſe Center Q muſt be found (as in *Fig.* 27;) then from I and N, draw Lines through Q at Pleaſure, to *b* and *c*, which continue up to L and P, and middle A I and B N, by Lines through the interſecting Arches T R S V, and where they cut the Lines L *c* and P *b*, as at W and X, is the Centers for the Curve A I B N, *&c.*

and

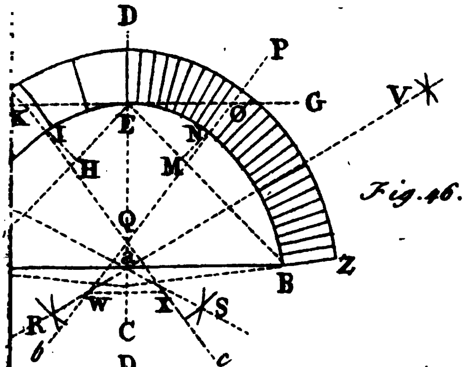


Fig. 46.

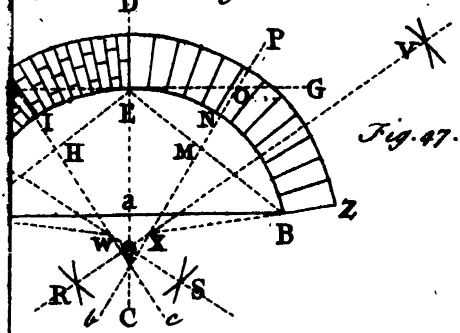


Fig. 47.

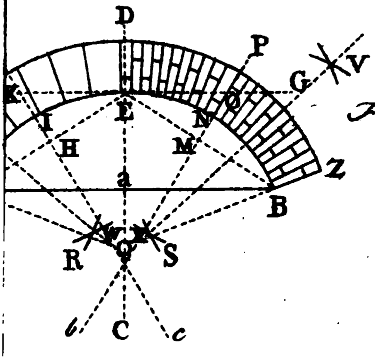


Fig. 48.

Fig. 49.

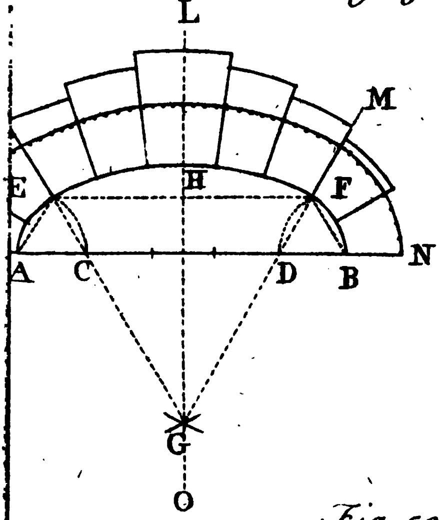
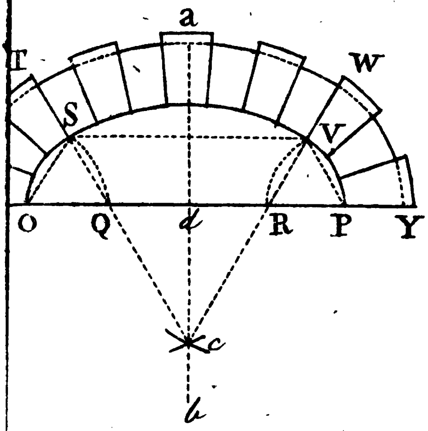


Fig. 50.



and the Joints of the Courſes between I N, depends on the Center Q, and thoſe between A I and BN, depends on the Centers W and X.

PLATE XIV.

To deſcribe the Centers and Summering of an Ellipſis-Arch, by equilateral Triangles.

See Fig. 49. Draw the Baſe A B, which middle by O L, of a Length at Pleaſure, and ſet on the Thickneſs of the Arch A I, and B N; then divide A B into five equal Parts, and draw two equilateral Triangles A E C and D F B, and from D ſtrike the Curve F B; alſo from C ſtrike the Curve A E, and through E C and F D, draw the Lines K G and M G, which meets at G in the Line O L, and is the Center of the Curve E H F, and all the other Curves and Summerings between EK and FM, and C is the Center of all thoſe between E K and I A, and D is the ſame to thoſe between F M and BN, and F G E F is an equilateral Triangle; becauſe two of the ſame Dimenſion, being ſet on a Line at any Diſtance, ſuppoſe C D and their Sides E C and F D, continued their Meeting as at G, will be an Angle whoſe Compliment is equal to thoſe at E and F.

To deſcribe another Ellipſis Arch, whoſe Baſe is divided into 4 equal Parts.

See Fig. 50. And obſerve OQ=Q*d*=*d*R, and RP are equal, then proceed in all other Reſpects as in the foregoing Example, Q is the Center of the Curves and Summerings between XO and ST, and R the Center of thoſe between VW and PY, and *c* the ſame to thoſe between ST and VW, and OSQ RVP and CSV are equilateral Triangles.

PLATE XV.

To deſcribe an Ellipſis Arch, whoſe Baſe is divided into three equal Parts.

See Fig. 51. AC, CD and DB are equal, and C is the Center of the Curves and Summerings between AK and EH, and D is the Center of thoſe between BZ and FI, and G of thoſe between EH and FI, and AEC, DFB, and GEF are equilateral Triangles.

To deſcribe an Ellipſis, and four different Gothick Arches on one Baſe.

See Fig. 52. Divide MN into three equal Parts, and middle it by *l m*; then deſcribe two Triangles MQO, and PRN, and from O and P ſtrike the Curves QM and RN; continue

Fig. 51.

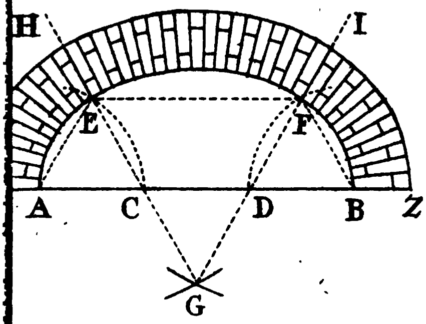
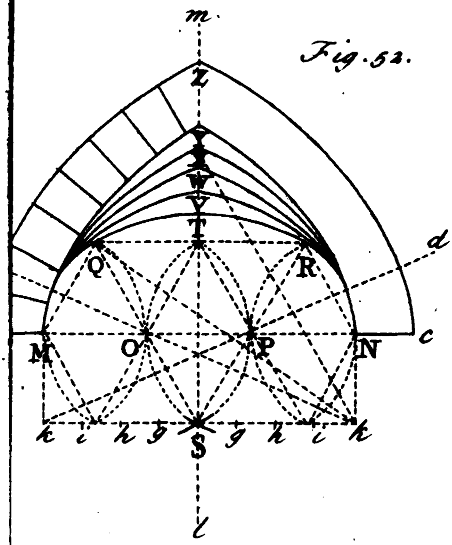


Fig. 52.



tinue Q O and R P to touch; *l m*, as at S, which is the Center of the Curve Q T R, and compleats the Elliptical-Arch MQTRN.

Obſerve the Method of deſcribing the laſt Arch, whereon the ſummering Courſes are laid down, and by which all the other may be deſcrib'd; draw Lines from the Centers *k k* through the Centers O and P to *b* and *d*, then is O the Center of the Curve and ſummering Joints between *a* M and *b e*, and P the Centers of *f* N and C *d*, and *k k* the Centers of the Curves and ſummering Joints between *e b*, and Y Z, and *f d*, and ſo of all Arches of this Kind, by drawing Lines to correſpond with both Centers.

PLATE XVI.

To deſcribe a ramping Ellipſis-Arch, by interſecting Lines and Centers.

See Fig. 53. Draw the horizontal Line A *a*, alſo the Baſe S T; on which deſcribe ABCD, middle A *a* and raiſe the Perpendicular E F, and divide A B, B F, F C and C D, each into the ſame Number of equal Parts, and draw the interſecting Lines, which gives the Curve A F D.

Then find the Centers for the outward Curve, and Croſs-Joints, (which muſt be parallel to the inward Curve,) draw the Line G H thro' F, at right Angles with B C, and find a Center on the Baſe-Line A D, to anſwer part of the Curve as I, which is the Center of A *b* and S K, with their Croſs-Joints; then thro' *b* and I, draw K L, and L is the Center of *b* F and K G, with their Croſs-Joints; then find the Center to part of the inward Curve, as from D to *d*, as at M which will not cut the Baſe Line S T, (it being ſo quick in the Haunch) from M ſtrike O T, and their Croſs-Joints; again, find the Center of another Part of the Curve as *c d*; which is at N, and is alſo the Center of Q O, and their Croſs-Joints; then through *d*, draw N O, and through N and *c* draw Q R, and R is the Center to F C and G Q, with their Croſs-Joints.

N. B. All ſummering Joints, muſt point to their Centers, except it be above the Baſe-Line as M; therefore, thoſe between O T, muſt point to ſome Part of the Baſe-Line, as to N; otherwiſe the Springing-Joint D T, will not range with S A.

To deſcribe an Egg-moulded-arch, by Interſecting Lines and Centers.

See Fig. 54. Draw the Baſe *l m*; alſo V *w* *x* Y; then middle V Y, and raiſe the Perpendicular *z a*, and divide V *w*, *w a*, *a x* and *x* Y, each into the ſame Number of equal Parts, and draw the interſecting Lines, which compleats the inward Curve V *n a b* Y.

Then

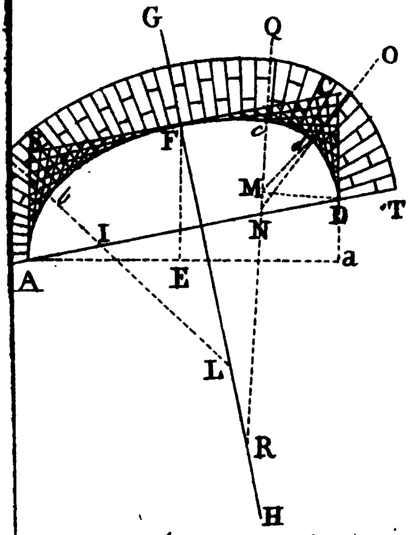


Fig. 53.

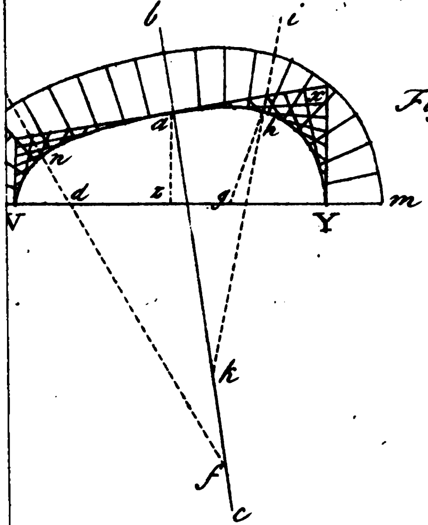


Fig. 54.

Then find the Centers of its ſeveral Parts that the outward Curve may be parallel thereto; firſt, through *a* draw *b c* at right Angles with *w x*, and *d* will be the Center of V *n*; then through *n* and *d*, draw *e f*, and *f* is the Center of *n a*; again, *g* is found to be the Center of Y *h*, draw *h g*; then find the Center of *a h*, which is at *k*; through *h*, draw *i k*, then from the ſeveral Centers ſtrike the outward Curves, and from *d* ſtrike *l e*, from *f* ſtrike *e h*, from *k* ſtrike *h i*, and from *g* ſtrike *i m*, then ſet on the ſeveral Courſes, and deſcribe the ſummering Joints tending to their proper Centers.

FINIS.

ERRATA.

Page 17. Line 7. for *z e*, *e x*, read Z *a*, *a* X. *Ibid.* Line 9. for *y z e x*, read Y Z *e* X. *Ibid.* Line 10. for *c b v c*, read *b* V C. *Ibid.* Line 19. for *w b*, read W *b*. *Ibid.* Line 22. for R Z *e s*, read R Z *e* S. *Ibid.* Line 26. for C *b v* C, read *b* V *c*. Page 18. Line 3. in *a* T *d a*, dele the laſt *a*. *Ibid.* Line 4. for *a* W *d a*, read *a* W *d*. *Ibid.* Line 11. for *d w*, read *d* W. Line *ibid*, for W C. read W *e*. Page 23. Line 15. for A I G H I B, read A ɪ G H ɪ B.

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